Class 20 – Exponential and Logarithmic Equations

Review

Definition: $b>0,\ b\neq 1$. $\boxed{y=\log_b x \iff b^y = x}$

Ex1: $64^{\frac{1}{2}} = \sqrt[2]{64^1}$ or $\sqrt{64} = 8$

Ex2: $\log_f(x-6) = 4 \iff f^4 = x-6$

Ex3: $4^{5k} = M \iff 5k = \log_4 M$

→ Exponential Equations : $\boxed{b^u = b^v \iff u=v}$

Ex1: $\sqrt{5} = 25^x \iff 5^{\frac{1}{2}} = (5^2)^x \iff 5^{\frac{1}{2}} = 5^{2x} \iff \frac{1}{2} = 2x \iff \boxed{x=\frac{1}{4}}$

Ex2: $6^{x-3} = \frac{1}{216} \iff 6^{x-3} = \frac{1}{6^3} \iff 6^{x-3} = 6^{-3} \iff x-3=-3 \iff \boxed{x=0}$

After comparing exponents: linear, quadratic, rational eqn's

Ex3: $2^{8x^2+7x-10} = 2^{17-8x} \iff \underset{+8x\ -17}{8x^2+7x-10} = \underset{-17\ +8x}{17-8x} \iff 8x^2+15x-27=0$

prod: -216
sum: $+15$

$8\cdot 27 = 216$

$\boxed{2,4 \quad 3,9}$

$2\cdot 108 = 216$
$4\cdot 54 = 216$
$3\cdot 72 = 216$
$9\cdot 24 = 216$

-9 & 24

$\underbrace{8x^2+24x}\underbrace{-9x-27}=0$

$8x(x+3) - 9(x+3) = 0$

$\underbrace{(x+3)}_{A}\underbrace{(8x-9)}_{B} = 0$

$x+3=0$ or $8x-9=0$

$\boxed{x=-3}$

$8x = 9$

$\boxed{x=\frac{9}{8}}$

Logarithmic Equation: $\boxed{\log_b u = \log_b v \iff u = v}$

Ex 4: $\log_3\left(\frac{x-12}{x+7}\right) = \log_3\left(\frac{x-6}{x+2}\right) \iff \frac{x-12}{x+7} = \frac{x-6}{x+2}$

Restricted values $\leftrightarrow$ den. $=0$: $x=-7$ and $x=-2$

$\frac{x-12}{\cancel{x+7}}\cancel{(x+7)}(x+2) = \frac{x-6}{\cancel{x+2}}(x+7)\cancel{(x+2)} \Rightarrow (x-12)(x+2) = (x+7)(x-6) \Rightarrow$

$x^2 - 10x + 24 = x^2 + x - 42 \iff -10x + 24 = x - 42 \iff -11x = -18 \iff \boxed{x = \frac{18}{11}}$

$-x^2 \qquad -x^2 \qquad -x \quad -24 \quad -x \quad -24$

$\frac{x-12}{x+7} > 0$ & $\frac{x-6}{x+2} > 0$ for $x = \frac{18}{11}$: $\frac{\frac{18}{11} - 12}{\frac{18}{11} + 7} = \frac{\frac{-114}{\cancel{11}}}{\frac{95}{\cancel{11}}} = \frac{-114}{95} < 0$

No solution, since the #'s in the the logarithm are not postive.

---

§ 5.2 Exponential & log equations

→ Use calculator to compute:

Ex1: $\frac{10e^2 - 2}{e^2 - 4} \approx 21.2126$

Ex2: $\frac{\ln 2 + \ln 11}{\ln 11} \approx 1.2891$

On CALCULATOR:

(10 e ^ (2) - 2) / (e ^ (2) - 4)

(ln(2) + ln(11)) ÷ ln 11

Ex3: $8^x = 32$

$2^{3x} = 2^5$

$3x = 5$

$\boxed{x = \frac{5}{3} = 1.666...}$

$8^x = 31$ (apply ln both sides)

P.R.

$\ln(8^x) = \ln(31)$ (apply power rule)

$\frac{x \cdot \cancel{\ln 8}}{\cancel{\ln 8}} = \frac{\ln 31}{\ln 8} \iff \boxed{x = \frac{\ln 31}{\ln 8}}$

$\boxed{x \approx 1.6531}$

Ex4: $5^x = 41^{x-6}$

Step 1: apply LN: $\ln(5^x) = \ln(41^{x-6})$

Step 2: apply P.R.: $x \cdot \ln 5 = (x-6) \cdot \ln 41$

Step 3: solve Lin. eqn: $x \ln 5 = x \ln 41 - 6 \ln 41$

$-x\ln 41 \quad -x\ln 41$

$x\ln 5 - x\ln 41 = -6\ln 41$

$\frac{x(\cancel{\ln 5 - \ln 41})}{\cancel{\ln 5 - \ln 41}} = \frac{-6\ln 41}{\ln 5 - \ln 41}$

$\boxed{x = \frac{-6\ln 41}{\ln 5 - \ln 41}}$

$\boxed{x \approx 10.5896}$

Ex5: $7^{2x+5} = 81$

Step 1 - Apply LN: $\ln(7^{2x+5}) = \ln 81$

Step 2 - Apply P.R.: $(2x+5)\ln 7 = \ln 81$

Step 3 - Solve eqn: $2x \cdot \ln 7 + 5 \cdot \ln 7 = \ln 81$

$-5\ln 7 \quad -\ln 7$

$\frac{2x \ln 7}{2\ln 7} = \frac{\ln 81 - \ln 7}{2\ln 7}$

$\boxed{x = \frac{\ln 81 - \ln 7}{2\ln 7}}$

$\boxed{x \approx -1.3708}$

Ex6: $5^{10x-7} = 21^{3-5x} \overset{1^{st}}{\Longleftrightarrow} \ln\left(5^{10x-7}\right) = \ln\left(21^{3-5x}\right)$ apply LN

Apply Power Rule $\overset{2^{nd}}{\Longleftrightarrow} (10x-7)\cdot\ln 5 = (3-5x)\ln 21$

Add $5x\ln 21$ both sides $\Longleftrightarrow 10x\ln 5 - 7\ln 5 = 3\ln 21 - \cancel{5x\ln 21}$
$+5x\ln 21 \qquad\qquad +\cancel{5x\ln 21}$

Add $7\ln 5$ both sides $\Longleftrightarrow 10x\ln 5 + 5x\ln 21 - \cancel{7\ln 5} = 3\ln 21$
$+\cancel{7\ln 5} \qquad +7\ln 5$

Factor x $\Longleftrightarrow 10x\ln 5 + 5x\ln 21 = 3\ln 21 + 7\ln 5$

Divide by $10\ln 5 + 5\ln 21$ both sides $\Longleftrightarrow \dfrac{x\cancel{(10\ln 5 + 5\ln 21)}}{\cancel{10\ln 5 + 5\ln 21}} = \dfrac{3\ln 21 + 7\ln 5}{10\ln 5 + 5\ln 21}$

Compute answer $\Longleftrightarrow \boxed{x = \dfrac{3\ln 21 + 7\ln 5}{10\ln 5 + 5\ln 21} \approx 0.6514}$

Log equations $\boxed{\log_b u = \log_b v \Longleftrightarrow u = v}$ $\boxed{y = b^x \Longleftrightarrow \log_b y = x}$

Ex1: $\log_5(x+5) = \log_5(8x-5) \Longleftrightarrow x+5 = 8x-5 \Longleftrightarrow \boxed{x = \frac{10}{7}}$

Ex2: $\log_4(4x-3) = 2 \to \log_4 y$: $2 = \log_4 y \Longleftrightarrow 4^2 = y \Longleftrightarrow y = 16$

$2 = \log_4 16$: $\log_4(4x-3) = \log_4(16) \Longleftrightarrow 4x-3 = 16 \Longleftrightarrow \boxed{x = \frac{19}{4}}$

Ex3: $\log_2(x+5) + \log_2(x+8) = 2\cdot\log_2 2$: $2\cdot\log_2 2 = \log_2(2^2) = \log_2 4$

$\log_2(x+5) + \log_2(x+8) = \log_2 4 \qquad 2 = \log_2 y \Longleftrightarrow 2^2 = y \Longleftrightarrow y = 4$

$\log_2((x+5)(x+8)) = \log_2 4 \Longleftrightarrow (x+5)(x+8) = 4 \Longleftrightarrow x^2+13x+40 = 4$

$\Longleftrightarrow x^2+13x+36 = 0 \Longleftrightarrow (x+4)(x+9) = 0$ $\Bigg\}$ $\boxed{x = -4}$ $\cancel{x = -9}$

Ex 4: $\ln 10 + \ln\left(\frac{x^2+9x}{10}\right) = 0$

$\left[0 = \ln y \iff e^0 = y \iff y = 1 \;\&\; 0 = \ln 1\right]$

$\ln 10 + \ln\left(\frac{x^2+9x}{10}\right) = \ln 1$

$\ln\left(\cancel{10} \cdot \frac{x^2+9x}{\cancel{10}}\right) = \ln(1)$

$\ln(x^2+9x) = \ln(1)$

$x^2+9x = 1$

$x^2+9x-1 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-9 \pm \sqrt{81-4\cdot 1\cdot(-1)}}{2\cdot 1}$

$\boxed{x = \frac{-9 \pm \sqrt{85}}{2}}$

$85 = 5 \cdot 17$

$\frac{x^2+9x}{10} > 0$: $\frac{\left(\frac{-9+\sqrt{85}}{2}\right)^2 + 9\left(\frac{-9+\sqrt{85}}{2}\right)}{10}$.

Ex5: $\ln(x) - \ln(x+5) = 6$ $\quad \left(6 = \ln y \iff e^6 = y \iff 6 = \ln e^6\right)$

$\ln x - \ln(x+5) = \ln(e^6)$

$\ln\left(\frac{x}{x+5}\right) = \ln(e^6)$

$\cancel{(x+5)}\frac{x}{\cancel{x+5}} = e^6(x+5)$

$x = x\cdot e^6 + 5e^6$

$-xe^6 \quad -xe^6$

$x - xe^6 = 5e^6$

$\frac{x(1-\cancel{e^6})}{1-\cancel{e^6}} = \frac{5e^6}{1-e^6}$

$\boxed{x = \frac{5e^6}{1-e^6}}$

$\cancel{\boxed{x \approx -0.0223}}$

NO SOLUTION

Ex6: $5 \cdot 125^{x-1} = 625^{2x+1}$

$5^1 (5^3)^{x-1} = (5^4)^{2x+1}$

$5^1 \cdot 5^{3(x-1)} = 5^{4(2x+1)}$

$5^1 \cdot 5^{3x-3} = 5^{8x+4}$

$5^{3x-2} = 5^{8x+4}$

$3x-2 = 8x+4$

$\boxed{x = -\frac{6}{5}}$

$5 = 5^1$

$125 = 5^3$

$625 = 5^4$